

Central jet production as a probe of the perturbative formalism for exclusive diffraction

V.A. Khoze^{1,2,a}, A.D. Martin¹, M.G. Ryskin^{1,2}

¹ Department of Physics and Institute for Particle Physics Phenomenology, University of Durham, Durham, DH1 3LE, UK

² Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg, 188300, Russia

Received: 11 May 2006 / Revised version: 3 July 2006 /

Published online: 24 October 2006 – © Springer-Verlag / Società Italiana di Fisica 2006

Abstract. We propose a new variable, R_j , in order to identify exclusive double-diffractive high E_T di-jet production. The variable R_j is calculated using the transverse energy E_T and pseudorapidity of the jet with the *largest* E_T . For a purely exclusive event the value of $R_j \rightarrow 1$, if we were to neglect hadronisation and the detector resolution effects. To illustrate the expected R_j distribution we also compute exclusive three-jet production; and, moreover, we include jet smearing effects. By studying the predictions as a function of the size of the rapidity interval, $\delta\eta$, which allows for additional gluon radiation, one can probe the QCD radiation effects which are responsible for the Sudakov suppression of the exclusive amplitude. In this way we may check and improve the formalism used to predict the cross sections of exclusive double-diffractive Higgs boson (and/or other new physics) production.

1 Introduction

Diffractive processes offer a unique means to discover new physics at the LHC; see for example [1–4]. An exciting possibility is to search for Higgs bosons in an exclusive reaction, that is, $pp \rightarrow p + H + p$, where the plus signs denote large rapidity gaps. This process allows for detailed measurements of the Higgs boson properties in an exceptionally clean environment and provides a unique signature, especially for the MSSM Higgs sector [5, 6]. In particular, the Higgs mass and spin–parity determination can be done irrespective of the decay mode, and these studies are at the heart of the recent proposal [7] to complement the central detectors at the LHC by forward proton taggers placed far away from the interaction point. However, the expected event rate is limited; it is strongly suppressed, in particular by a Sudakov form factor necessary to guarantee the exclusive final state; see for instance [8, 9]. An analogous Sudakov suppression enters the predictions for the exclusive production of di-jets, $\gamma\gamma$, etc. The existing diffractive Tevatron data (see, for example, the reviews [10–15] and references therein) are not in disagreement with the theoretical expectations for these processes [16–20]. However a definitive¹ confirmation of the mechanism of central diffractive production is still desirable.

Here we examine in more detail the prediction for the important process of central diffractive di-jet production at the Tevatron. This process is a valuable luminosity monitor for central diffractive Higgs production, and for other exclusive processes which may reveal new physics, at the LHC. The corresponding cross section was evaluated to be about 10^4 times larger than that for the SM Higgs boson. Thus, in principle, the exclusive production of a pair of high E_T jets (that is, $p\bar{p} \rightarrow p + jj + \bar{p}$ in the case of the Tevatron) appears to be an ideal ‘standard candle’ for the Higgs boson. Note that the CDF measurements have already started to reach values of the invariant mass of the pomeron–pomeron system in the SM Higgs mass range. This process is important on its own right as a gluon factory. As discussed in [2, 23] the remarkable purity of the diffractively produced di-gluon system would provide a unique environment to study the properties of high energy gluon jets. Unfortunately, in the present CDF experimental environment, which does not provide tagging of both forward protons, the separation of exclusive events is not completely unambiguous. In particular, in addition to the smearing due to the jet-searching algorithm and detector effects (see for example [24, 25]), there are also hadronisation and QCD radiative effects, which

^a e-mail: V.A.Khoze@durham.ac.uk

¹ The observation of exclusive χ_c and $\gamma\gamma$ events [14, 15, 21] by the CDF collaboration has been reported at the conferences. These results appear to be consistent with the perturbative QCD expectations [18, 22], though in reality the scale of the χ_c production process is too low to justify the use of the perturba-

tive QCD formalism. The Tevatron exclusive $\gamma\gamma$ data are very important. Here we do not face problems with hadronisation or with the identification of the jets. However the exclusive cross section is rather small. Future precise measurements in the di-photon mass interval 10–20 GeV would allow a significant reduction to be realised of the uncertainties in the expectations for Higgs production, to the order of 30%–50%.

distort the manifestation of the exclusive di-jet signal; see for example [20, 26]. Because the reliability of the predictions for the cross sections of central exclusive production of heavy mass objects is so important for the prospects of forward physics studies at the LHC, it is pivotal to check (whenever possible) all the important ingredients of the perturbative QCD approach derived in [2, 8]. In this paper we focus on how to expose the role of the crucial QCD radiative effects which regulate the amount of Sudakov suppression.

Recall that already in QED it is well known that we can never observe a pure exclusive process. For example, the cross section for $e^-e^+ \rightarrow \mu^-\mu^+$ is exactly zero, if we exclude the photon radiation and additional lepton-pair production which may accompany such events; for a review, see [27]. To determine the cross section we must use the celebrated Bloch–Nordsieck [28] and Kinoshita–Lee–Nauenberg [29, 30] theorems, and calculate the radiative correction accounting for the experimental resolution. In experiments with very good resolution the corrections are quite large.

An analogous situation occurs when we consider QCD exclusive processes. Here we will apply the Bloch–Nordsieck procedure to exclusive diffractive di-jet production. That is, we will allow for additional gluon radiation in some rapidity interval $\delta\eta$, and study how the cross section changes as we change the size of $\delta\eta$ and the energy fraction which is allowed to radiate into $\delta\eta$. At present, two extreme mechanisms are used to describe central diffractive di-jet production. First, the formalism for pure exclusive production [8] has been implemented in the ExHuMe Monte Carlo package [31]. Second, central inelastic di-jet production via the inelastic interaction of two soft pomerons, which results in parton–parton scattering at large E_T ; this process is implemented in the POMWIG Monte Carlo package [32]. The di-jet distribution is plotted in terms of the variable

$$R_{jj} = M_{jj}/M_X. \quad (1)$$

In terms of this variable, the first process corresponds to $R_{jj} = 1$, since the mass of the di-jet system, M_{jj} , is equal to the mass, M_X , of the whole central system. The second process has $R_{jj} < 1$ since additional radiation (the fragments of the pomerons) populate the central region, that is, $M_X > M_{jj}$.

2 A new signature R_j of exclusive di-jet events

Di-jet production, with a rapidity gap on either side, has been measured by the CDF collaboration, both in Run I [33] and in Run II [11–15], at the Tevatron. However there may still be some room for doubt whether *exclusive* di-jet production, $p\bar{p} \rightarrow p + jj + \bar{p}$, has been actually observed. As mentioned above, there are various effects which strongly smear the R_{jj} distribution, especially in the absence of double proton tagging. The hope was that

exclusive events would show up as a peak at $R_{jj} = 1$. Unfortunately the R_{jj} distribution is strongly smeared out by QCD bremsstrahlung, hadronisation, the jet-searching algorithm and other experimental effects. For example, it was shown, using the ExHume Monte Carlo package [34], that only about 10% of exclusive events with $E_T > 7$ GeV have finally $R_{jj} > 0.8$, with the CDF cuts used in Run I at the Tevatron.

To weaken the role of this smearing we propose to measure the di-jet distribution in terms of a new variable

$$R_j = 2E_T(\cosh \eta^*)/M_X, \quad (2)$$

where only the transverse energy E_T and the rapidity η of the jet with the *largest* E_T are used in the numerator. Here $\eta^* = \eta - Y_M$ where Y_M is the rapidity of the whole central system². Clearly the jet with the largest E_T is less affected by hadronisation, final parton radiation etc. In particular, final state radiation at the lowest order in α_s will not affect R_j at all, since it does not change the kinematics of the highest E_T jet used to evaluate (2). So despite the emission of an extra jet during the final parton shower, we still have $R_j = 1$. Thus, to see the role of QCD radiation on the R_j distribution, we only account explicitly for additional gluon radiation in the initial state. At leading order, it is sufficient to consider the emission of a third gluon jet, as shown in Fig. 1. The reason why it is sufficient to consider only one extra jet is that the effect of the other jets, which, at LO, carry lower energy due to the strong ordering, is almost negligible in terms of the R_j distribution. The rapidity Y_M is sketched in Fig. 2. In Sect. 5 we will compute the exclusive three-jet cross section for different choices of the rapidity interval $\delta\eta$ containing the jets.

3 Resumé of the calculation of exclusive di-jet production

To compute the R_j distribution we first calculate the cross section of the exclusive di-jet production of Fig. 1. We have $\sigma_{\text{excl}} = \mathcal{L}\hat{\sigma}$, where [8]

$$\mathcal{L} \simeq \frac{\hat{S}^2}{b^2} \left| \frac{\pi}{8} \int \frac{dQ_t^2}{Q_t^4} f_g(x_1, x'_1, Q_t^2, \mu^2) f_g(x_2, x'_2, Q_t^2, \mu^2) \right|^2. \quad (3)$$

The first factor, \hat{S}^2 , is the probability that the rapidity gaps survive against population by secondary hadrons from the underlying event, that is, hadrons originating from *soft* rescattering. It is calculated using a model which embodies all the main features of soft diffraction [35]. It is found to be $\hat{S}^2 = 0.026$ for $pp \rightarrow p + H + p$ at the LHC. The remaining factor, $|\dots|^2$, however, may be calculated using perturbative QCD techniques, since the dominant contribution

² Note that we systematically neglect the effects arising from the transverse momentum of the di-jet system, which is very small compared to the E_T resolution.

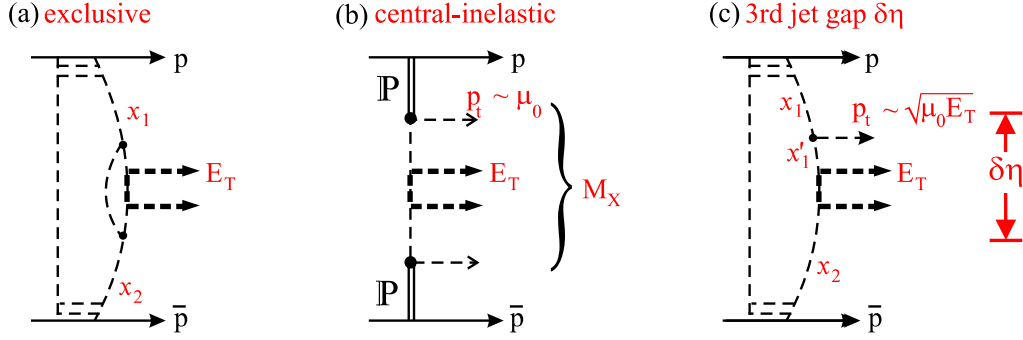


Fig. 1. Central diffractive di-jet production; **a** purely exclusive, **b** via soft pomeron–pomeron interactions, and **c** with a third jet in a given rapidity interval $\delta\eta$. The *dashed lines* represent gluons

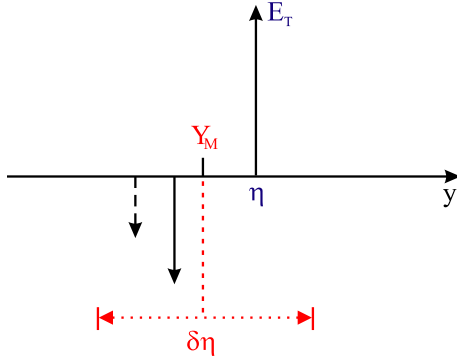


Fig. 2. The rapidity Y_M of the central system. It does not necessarily occur at $y = 0$. The rapidity interval containing the jets is denoted by $\delta\eta$, outside of which there is no hadronic activity

to the integral comes from the region $\Lambda_{\text{QCD}}^2 \ll Q_t^2 \ll M_H^2$. The probability amplitudes, f_g , to find the appropriate pairs of t -channel gluons (Q, q_1) and (Q, q_2), are given by the skewed unintegrated gluon densities at a *hard* scale $\mu \sim M_H/2$.

Since the momentum fraction x' transferred through the screening gluon Q is much smaller than that (x) transferred through the active gluons ($x' \sim Q_t/\sqrt{s} \ll x \sim M_H/\sqrt{s} \ll 1$), it is possible to express $f_g(x, x', Q_t^2, \mu^2)$ in terms of the conventional integrated density $g(x)$. A simplified form of this relation is [8]

$$f_g(x, x', Q_t^2, \mu^2) = R_g \frac{\partial}{\partial \ln Q_t^2} \left[\sqrt{T_g(Q_t, \mu)} x g(x, Q_t^2) \right], \quad (4)$$

which holds to 10%–20% accuracy. The factor R_g accounts for the single log Q^2 skewed effect. It is found to be about 1.4 at the Tevatron energy and about 1.2 at the energy of the LHC.

Note that the f_g 's embody a Sudakov suppression factor T , which ensures that the gluon does not radiate in the evolution from Q_t up to the hard scale $\mu \sim M_H/2$, and so it preserves the rapidity gaps. The Sudakov factor

is [8, 36, 37]

$$T_g(Q_t, \mu) = \exp \left(- \int_{Q_t^2}^{\mu^2} \frac{\alpha_s(k_t^2)}{2\pi} \frac{dk_t^2}{k_t^2} \times \left[\int_{\Delta}^{1-\Delta} z P_{gg}(z) dz + \int_0^1 \sum_q P_{qg}(z) dz \right] \right), \quad (5)$$

with $\Delta = k_t/(\mu + k_t)$. The square root arises in (4) because the (survival) probability not to emit any additional gluons is only relevant to the hard (active) gluon. It is the presence of this Sudakov factor which makes the integration in (3) infrared stable and perturbative QCD applicable.

It should be emphasised that the presence of the double logarithmic T -factors is a purely classical effect, which was first discussed in 1956 by Sudakov in QED [38]. There is strong bremsstrahlung when two colour charged gluons ‘annihilate’ into a heavy neutral object and the probability not to observe such a bremsstrahlung is given by the Sudakov form factor. Therefore, any model (with perturbative or non-perturbative gluons) must account for the Sudakov suppression when producing exclusively a heavy neutral boson via the fusion of two coloured/charged particles.

In fact, the T -factors can be calculated to *single* log accuracy [5]. The collinear single logarithms may be summed up using the DGLAP equation. To account for the ‘soft’ logarithms (corresponding to the emission of low energy gluons) the one-loop virtual correction to the $gg \rightarrow H$ vertex was calculated explicitly, and then the scale $\mu = 0.62M_H$ was chosen in such a way that (5) reproduces the result of this explicit calculation [5]. It is sufficient to calculate just the one-loop correction since it is known that the effect of ‘soft’ gluon emission exponentiates. Thus (5) gives the T -factor to single log accuracy³.

³ Of course, in the case of QCD, the exponentiation of soft emission requires some clarification. Because of the non-Abelian structure of QCD, there are indeed some particular cases when the soft-emission factorisation and Poisson distribution theorems do not hold. This was exemplified, in particular, in [39]. However we are interested in a phenomenon of a com-

4 Calculation of exclusive three-jet production

Here we consider the emission of a third jet described by the variables x and p_t . The variable x is the fraction of the momentum of the incoming gluon (denoted by x_1 in Fig. 1c) carried by the third, relatively soft, jet; that is, $x = 1 - x'_1/x_1$. The explicit formula for the LO third jet radiation can be obtained using the helicity formalism reviewed in [41]. We outline the calculation in the appendix, where the general formulae for the exclusive three-jet production amplitude are presented; that is, not restricted to LO. In the double logarithm limit, with $p_t \ll E_T$ and $x \ll 1$, the exclusive three-jet cross section is simply the exclusive di-jet cross section, $\hat{\sigma}^{(2)}$, multiplied by the classical probability for soft gluon emission

$$d\hat{\sigma}_{\text{LO}}^{(3)} = d\hat{\sigma}^{(2)} \frac{1}{4} \left(\frac{N_c \alpha_s}{\pi} \frac{dp_t^2}{p_t^2} \frac{dx}{x} \right). \quad (6)$$

Note the extra factor $1/4$, which reflects the suppression of soft gluon emission in comparison with the usual classical result given by the expression in brackets. Naively we might expect a colour factor N_c , but instead we have $N_c/4$. This is due to the absence of the colour correlation between the left (amplitude M) and the right (amplitude M^*) parts of the diagram for the cross section, in our case with a colour-singlet s -channel state.

If we just keep the collinear logs with respect to the beam direction, that is, we keep the condition $Q_t < p_t \ll E_T$, but do not impose $x \ll 1$, then the three-jet cross section becomes

$$\frac{d\hat{\sigma}_{\text{LO}}^{(3)}}{dt} = \hat{\sigma} \left(\frac{N_c \alpha_s}{4\pi} \frac{dp_t^2}{p_t^2} \frac{dx}{x} \right), \quad (7)$$

where

$$\hat{\sigma} = \left(\frac{9\pi\alpha_s^2 (E_T^2)}{4E_T^4} \right) \frac{1}{2} \left[(1-x)^3 + \frac{1+x^4(1-2E_T^2/M_{\text{jj}}^2)}{1-x} \right]. \quad (8)$$

The first term, in the round brackets in (8), is the known cross section for the exclusive colour-singlet gg -di-jet production. The variable t in (7) denotes the square of the 4-momentum transferred in this exclusive colour-singlet $gg \rightarrow$ high E_T di-jet process. In other words t is measured between the highest E_T jet and the incoming gluon which produces the high E_T di-jet system. The last term

pletely different (classical) nature. In [5] we discussed the NLO correction to the double log term caused by the *classical current*, where the soft gluon radiation exponentiates. This accounts for the effect of the energy- and angular-ordered additional soft gluon radiation, which, due to QCD coherence, is just part of the cascade generated by the ‘primary’ gluon. Summation of such soft ‘single’ logs is performed analogously to the DGLAP approach, which results in their exponentiation. This situation is of the same nature as the well known modified leading logarithmic approximation, which, for example, is discussed in detail in the book by Dokshitzer et al. [40].

in round brackets in (7) is just the double log expression for the emission of the third jet; see (6). Finally, the factor in square brackets in (8) accounts for the polarisation structure of the three-jet system. Recall that the exclusive double-diffractive kinematics selects events with the same helicities of the incoming gluons, either $(++)$ or $(--)$; that is, $J_z = 0$. The first term, $(1-x)^3$, corresponds to the helicity of the soft (third) jet being equal to the helicities of the incoming gluons, whereas the remaining expression corresponds to the third jet having opposite helicity to that of the incoming gluons. In this expression, the term proportional to x^4 originates from the high E_T di-jets having different helicities, whereas the factor 1 in the numerator corresponds to the production of two high E_T jets with the helicities equal to each other. The $1/(1-x)$ in the second term reflects the usual (BFKL-like) $1/z$ singularity in the Altarelli–Parisi splitting function $P(z)$.

It is informative to note that the behaviour of all three terms in the square brackets of (8), in the $x \rightarrow 0$ or $x \rightarrow 1$ limits, is not accidental. Its physical origin can be understood by recalling the celebrated Low soft-bremsstrahlung theorem [42] (see also [43, 44]). Recall that according to the MHV rules (see the appendix), the only non-vanishing Born $2 \rightarrow 2$ amplitudes, M_B , are those which have two positive and two negative helicities. On the other hand, the $J_z = 0$ selection rule requires that the two incoming gluons have the same helicities, either $(++)$ or $(--)$. According to the Low theorem [42] for radiation of a soft gluon with energy fraction $z \ll 1$, the radiative matrix element M_{rad} may be expanded in powers of z ,

$$M_{\text{rad}} \sim \frac{1}{z} \sum_0^{\infty} C_n z^n, \quad (9)$$

where the first two terms, with coefficients C_0 and C_1 (which correspond to long-distance radiation), can be written in terms of the non-radiative matrix element M_B .

The application of these classical results is especially transparent when the cross sections are integrated over the azimuthal angles. Then the non-radiative process depends only on simple variables, such as the centre-of-mass energy⁴. In particular, if $M_B = 0$, the expansion starts from the non-universal $C_2 z^2$ term, which corresponds to non-classical (short-distance) effects, not related to M_B [43, 44].

Let us start with the third term in the square brackets in (8). In this case soft radiation should be considered with $z = x \ll 1$. The corresponding non-radiative matrix element vanishes, since its helicity structure is either $(+++-)$ or $(--++)$. Therefore, the matrix element squared, $|M_{\text{rad}}|^2$, is proportional to x^2 . Keeping in mind the factor $x^2 dx/x$, which arises from phase space, we see that this term is indeed proportional to $x^4 dx/x$, as it appears in (8). The soft-radiation limit of the first term corresponds to $z = (1-x) \ll 1$. Then the third jet carries the largest momentum, and one of the final jets is very soft. Again, the corresponding Born amplitude vanishes due to the MHV

⁴ Note that in our case, in the collinear log approximation, when $Q_t \ll p_t \ll E_T$, the azimuthal angular dependence is practically absent.

rule, and we arrive at the result $|M_{\text{rad}}|^2 \sim (1-x)^4 d(1-x)/(1-x)$. Finally, the second term, with the factor 1 in the numerator, corresponds to the only non-vanishing non-radiative amplitude, either $(+ + - -)$ or $(- - + +)$.

In the case of the collinear LO process (i.e. $p_t \ll E_T$), the value of R_j can be calculated as

$$R_j = \sqrt{1-x} \left(\frac{\cosh(\eta^*)}{\cosh(\eta^* \pm \frac{1}{2} \ln(1-x))} \right). \quad (10)$$

Here $\sqrt{1-x} = M_{\text{jj}}/M_X$ accounts for a lower mass, M_{jj} , of the di-jet system in comparison with the mass M_X of the three-jet system, whereas the factor in brackets accounts for the corresponding shift (by $0.5 \ln(1-x)$) of the rapidity of di-jet system. The minus sign must be used in (10) when the highest E_T jet goes in the same (beam or target) hemisphere as the soft (third) jet.

5 How the third jet affects the distribution in R_j

With knowledge of the luminosity, (3), and the cross section of the hard subprocess, (8), we can calculate the cross section of exclusive three-jet production, and study how this contribution looks in terms of the R_j variable. Note that, after the emission of the third jet, the production of other soft jets with $x' < x$ practically does not alter the value of R_j .

In the naive case, à la QED, this multi-jet emission cancels a large part of the Sudakov T -factor suppression. In other words, it gives an exponent analogous to that in (5), but with a positive power. In QCD the situation is more complicated. In the expression for the cross section, MM^* , the two active t -channel gluons (one in M , the other in M^*) are not correlated with each other, but form colour singlets, each with the corresponding screening gluon in its own amplitude, M or M^* ; see Fig. 3. The colour decomposition of the t -channel pair of active gluons, gg' , is given by

$$gg' = \sum_i c'_i A_i = \frac{1}{64} A_1 + \frac{8}{64} A_8 + \frac{8}{64} A_{\bar{8}} + \frac{10}{64} A_{10} + \frac{10}{64} A_{\bar{10}} + \frac{27}{64} A_{27}, \quad (11)$$

where A_i denotes the colour multiplet of the t -channel gg' system; that is, A_1 is the colour singlet, A_8 and $A_{\bar{8}}$ (A_{10} and $A_{\bar{10}}$) are the asymmetric and symmetric colour octets (decuplets) components, etc. The coefficients c'_i give the probability to have one or another colour state. Thus the probability that the pair of active gluons, gg' , forms the corresponding colour multiplet is

$$c_i \equiv ic'_i, \quad (12)$$

that is, c'_i times the statistical weight given by the number i of members of the multiplet.

If we use the decomposition of the product of two three-gluon vertices $i^2 f_{abe} f_{cde}$ over the colour projection operators P_i , that is

$$i^2 f_{abe} f_{cde} = \left(3P_1 + \frac{3}{2}P_8 + \frac{3}{2}P_{\bar{8}} - P_{27} \right)_{ab,cd}, \quad (13)$$

then we see that for each t -channel colour multiplet, the probability of soft gluon emission is driven by its own colour factor λ_i . Namely, we have $\lambda_1 = N_c = 3$ for the singlet, $\lambda_8 = 3/2$ for the octets, $\lambda_{10} = 0$ for the decuplets and $\lambda_{27} = -1$ for the 27-multiplet. The colour labels a, b, c, d, e are shown in Fig. 3.

So to compute the R_j distribution we must include the factors arising from including the third jet with the corresponding colour charge λ_i for each term in the decomposition (11). The power of the exponent for this real emission has the form of the T for the virtual corrections (5) multiplied by the corresponding colour factor λ_i/N_c . For instance, for the case when the gg' pair form a singlet, that is, for $i = 1$, we have $\lambda_i/N_c = 1$. Taking each exponent with its weight c_i , we obtain

$$T^{(\text{real})} = \sum_i c_i \exp \left(\frac{\lambda_i}{N_c} \int_{Q_t^2}^{\mu^2} \frac{\alpha_s(k_t^2)}{2\pi} \frac{dk_t^2}{k_t^2} \int_{\Delta}^x P_{gg}(z) dz \theta(\delta\eta/2 - |\eta|) \right), \quad (14)$$

where the scale $\mu = 0.62 \sqrt{M_{\text{jj}}^2/(1-x)}$ is taken to be the same as in (5) and where the coefficients $c_i = ic'_i$ are the weightings in the decomposition shown in (11). Unlike (5), the z integral is limited by the momentum fraction x carried by the soft third jet; for the case of $x > 1/2$ the upper limit x in the z integral (14) is replaced by $1-x$ – two jets cannot carry the fraction of an initial momentum greater than 1 (i.e. $x+z < 1$). Next, we have added the θ -function, which enables us to vary the size of the $\delta\eta$ interval containing the jets, so that we can study the radiation effect in more detail. As a rule, the jet reconstruction is performed in some limited rapidity interval, so it is natural to select

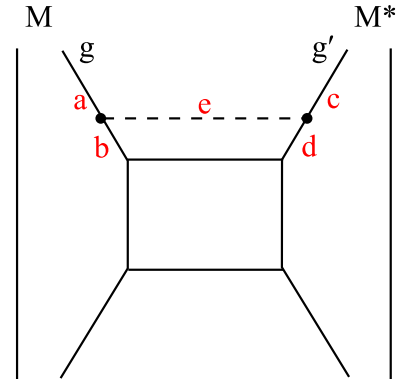


Fig. 3. The cross section, MM^* , for exclusive three-jet production, where the active gluons are denoted by g and g' ; see the t -channel decomposition of (11). The two outside vertical lines are the screening gluons; indeed all the lines in the plot denote gluons. The dashed line is the third (soft) jet, with kinematic variables x and p_t . The colour labels a, b, c, d, e are those used in (13)

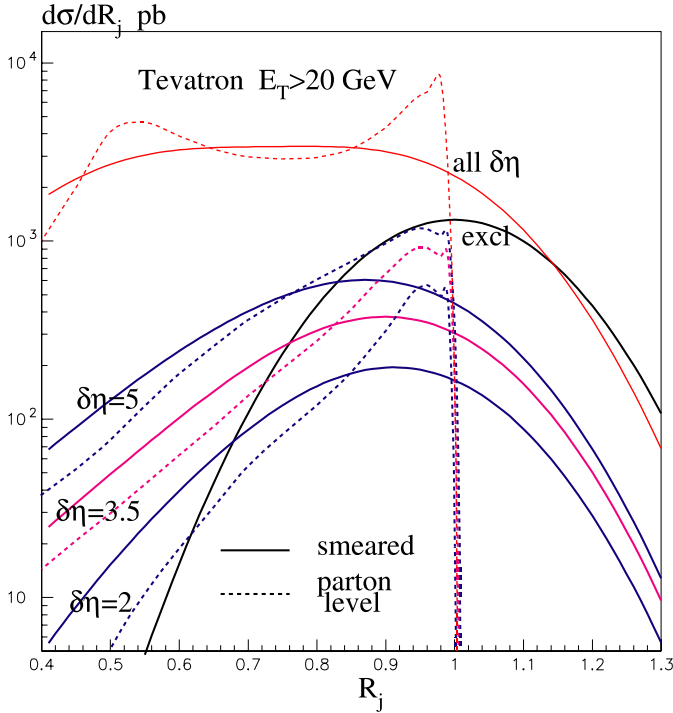


Fig. 4. The R_j distribution of exclusive two- and three-jet production at the Tevatron. Without smearing, exclusive two-jet production would be just a δ -function at $R_j = 1$. The distribution for three-jet production is shown for different choices of the rapidity interval, $\delta\eta$, containing the jets; these distributions are shown with and without smearing. The highest E_T jet must have $E_T > 20$ GeV

events where all the jets are emitted within the interval $\delta\eta$ centred at the position of the M_X system (that is in the interval $\pm\delta\eta/2$ in the frame where $Y_M = 0$; see Fig. 2), while any hadron activity outside the interval $\delta\eta$ is forbidden.

Note that, due to a more complicated colour structure in QCD, even in the double log limit, there is no exact cancellation between the real emission (14) and the Sudakov T -factor (5)⁵.

To calculate the exclusive cross section for three-jet production accompanied by the emission of softer jets in the rapidity interval $\delta\eta$, we multiply the exclusive luminosity (3) by the cross section of the hard (LO three-jet production) subprocess, (8), and by the factor $T^{(\text{real})}$, (14), to account for the allowed radiation of softer gluons. The results are presented in Figs. 4 and 5 in terms of distributions over the new variable R_j . In order to do this, relation (10) was used to transform the distributions over the momentum fraction x carried by the soft gluon, into the R_j distributions presented in the figures.

To be explicit the procedure is as follows. The R_j distribution is computed using

⁵ The simplest example of this lack of cancellation is exclusive Higgs boson production, where already at the first α_s order there is Sudakov suppression (5), while it is impossible to emit only one gluon accompanying the Higgs boson from the colourless two gluon state.

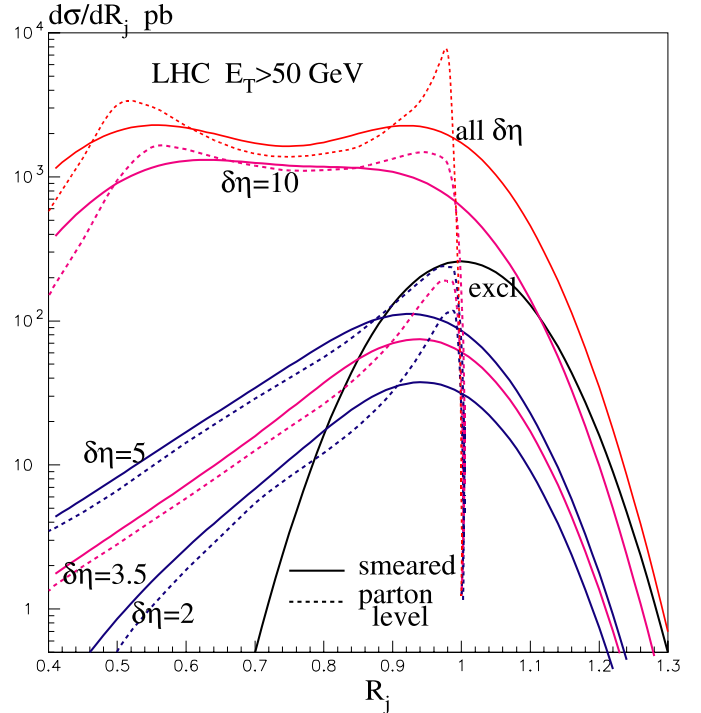


Fig. 5. The R_j distribution of exclusive two- and three-jet production at the LHC. Without smearing, exclusive two-jet production would be just a δ -function at $R_j = 1$. The distribution for three-jet production is shown for different choices of the rapidity interval, $\delta\eta$, containing the jets; these distributions are shown with and without smearing. The highest E_T jet must have $E_T > 50$ GeV

$$\frac{d\sigma}{dR_j} = \int dE_T^2 d\eta_1 d\eta_2 dp_t^2 \mathcal{L} \left(\frac{d\hat{\sigma}^{(3)}}{dt dp_t^2 dx} \right) \times (T^{(\text{real})})^2 \left(\frac{dR_j}{dx} \right)^{-1}, \quad (15)$$

where the luminosity \mathcal{L} is given in (3) and the Sudakov factor $T^{(\text{real})}$ is given by (14), and where η_1 and η_2 are the rapidities of the high E_T jets. We integrate over the kinematic intervals

$$E_T > E_{\min}, \quad |\eta_{1,2}| < 2.5, \quad p_{\min} < p_t < p_{\max}. \quad (16)$$

The lower limit of the logarithmic p_t integral is given either by the transverse momentum Q_t in the gluon loop⁶ or by the allowed rapidity interval $\delta\eta$; that is, $p_{\min} = \max \{ Q_t, xM_{Xe}^{-\delta\eta/2} \}$. The upper limit is of a pure kinematical nature: $p_{\max} = \min \{ E_T, xM_X/2 \}$. If $p_{\max} < p_{\min}$, then there is no LO contribution.

Next, we have to include the emission of the third (soft) jet in the direction of one or the other incoming gluons;

⁶ For $p_t < Q_t$, the destructive interference between emissions from the active gluon x_1 and from the screening gluon (that is, the left gluon in Fig. 1a and c) kills the logarithmic p_t integration. Strictly speaking the values of Q_t in the amplitudes M and M^* may be different, but this effect is beyond the LO accuracy of our calculation.

that is, beam protons. In other words we must sum up the contributions with either the plus or minus signs plus in (10). Thus, finally, we obtain

$$\begin{aligned} \frac{d\sigma}{dR_j} &= \frac{\hat{S}^2}{b^2} \int dE_T^2 d\eta_1 d\eta_2 \sum_{+,-} \sum_i c_i \hat{\sigma} \left(\frac{N_c \alpha_s}{4\pi} \right) \\ &\left| \frac{\pi}{8} \int \frac{dQ_t^2}{Q_t^4} f_g(x_1, x'_1, Q_t^2, \mu^2) f_g(x_2, x'_2, Q_t^2, \mu^2) \exp(n_i) \right. \\ &\times \left. \sqrt{\ln(p_{\max}^2/p_{\min}^2)} \right|^2 \left(x \frac{dR_j}{dx} \right)^{-1}, \end{aligned} \quad (17)$$

where n_i denotes the power in the exponent in $T^{\text{(real)}}$ of (14). The quantity $\hat{\sigma}$ arising from the hard $gg \rightarrow ggg$ subprocess is given by (8). Note that the factor dp_t^2/p_t^2 in (7) gives rise to the logarithm in \mathcal{L} in (17), while the factor dx/x goes into $(x dR_j/dx)^{-1}$. Indeed, the value of x and the derivative

$$\frac{dR_j}{dx} = \frac{R_j}{2(1-x)} \left[\pm \tanh \left(\eta^* \pm \frac{1}{2} \ln(1-x) \right) - 1 \right] \quad (18)$$

are calculated according (10). Note that since the lower limit, p_{\min} , of the integration over the p_t of the soft jet may depend on the transverse momentum, Q_t , in the internal gluon loop, the factors $\exp(n_i)$ and $\ln(p_{\max}^2/p_{\min}^2)$ occur inside the ‘luminosity Q_t^2 integral’.

In the computation we have used the partons of [45]. We neglect hadronisation effects and present the parton level results by dashed curves. In terms of the R_j distribution, the exclusive di-jet contribution occurs as a δ -function, $\delta(R_j - 1)$ and cannot be shown in the figures. However, in any realistic experiment, the distribution is smeared, at least by fluctuations in the calorimeter⁷. To see the effect of more or less realistic smearing, we assume a Gaussian distribution with a typical resolution⁸ $\sigma = 0.6/\sqrt{E_T}$ in GeV.

The results obtained, after this smearing of the parton level distributions, are shown by the continuous curves in Figs. 4 and 5. We see that for the case of $\delta\eta < 5$ the exclusive di-jet production still dominates for $R_j > 0.7-0.8$. The perturbative QCD radiation is suppressed by the extra coupling α_s . However this suppression is partly compensated by the collinear logs and by a large longitudinal phase

space; that is, by the rapidity interval $\delta\eta$ allowed for the emission of the extra soft jets. Indeed, we see that the cross section grows with $\delta\eta$, and by $\delta\eta > 10$ is close to the saturation curve (denoted ‘all $\delta\eta$ ’), which covers the whole interval of leading log QCD radiation.

Note that in the region $R_j < 0.6-0.7$ the dominant contribution comes from three-jet emission. Moreover here the results are more weakly dependent on possible smearing. Of course, in the region of small R_j there may be other contributions coming from the three- or four-jet Mercedes-like configurations⁹. However these contributions are not expected to be large, since in this case α_s is not compensated by large logs. Another possible contribution comes from configurations which look like inelastic di-jet production in the collisions of two soft pomerons. Such configurations, corresponding to Fig. 1b, may populate the low R_j region and are beyond the scope of the present analysis.

6 General use of R_j

In spite of the fact that the R_j variable was introduced to select exclusive di-jets in double-diffractive hadron–hadron interactions in which both of the outgoing protons are tagged, a similar idea can be used to improve the measurements of the light-cone momentum fraction carried by the di-jet system in other situations. In particular, to measure the fraction of the photon momentum, x_γ , carried by the high E_T di-jets in DIS. Note that the final state radiation (and hadronisation) affect mainly the energy, and much less the rapidity of the jet. Therefore to calculate x_γ (or x_{jj}^+ and x_{jj}^- in the more general case) one can use the E_T of the largest E_T jet together with the rapidity of each jet.

Acknowledgements. We thank Mike Albrow, Michele Arneodo, Andrew Brandt, Duncan Brown, Brian Cox, Albert De Roeck, Dino Goulianos, Risto Orava, Andy Pilkington and Koji Terashi for useful discussions. MGR would like to thank the IPPP at the University of Durham for hospitality, and ADM thanks the Leverhulme Trust for an Emeritus Fellowship. This work was supported by the Royal Society, the UK Particle Physics and Astronomy Research Council, by grants RFBR 04-02-16073, 07-02-00023 and by the Federal Program of the Russian Ministry of Industry, Science and Technology SS-1124.2003.2, and by INTAS grant 05-103-7515.

Appendix: Helicity amplitudes for $gg \rightarrow ggg$

Here we outline the formalism used to calculate the $gg \rightarrow ggg$ process shown in Fig. 6. We denote the colour indices of the incoming gluons by a, b , and of the outgoing high E_T gluons by c, d . Finally the colour index of the soft jet is denoted by e . The $gg \rightarrow ggg$ matrix element, which depends

⁷ If we assume that the two forward protons are tagged (as is possible, in principle, in the D0 experiment at the Tevatron [46, 47] or at the LHC if the CMS and/or ATLAS detectors are supplemented by the Roman Pots) then the mass of the whole system, M_X , can be measured with much better accuracy by the missing mass method.

⁸ We thank M.G. Albrow, D. Alton, M. Arneodo, A. Brandt, C. Buttar, R. Harris, C. Royon and K. Terashi for discussions on this choice. The resolution $\sigma = 0.6/\sqrt{E_T}$ in GeV is close to that obtained for the CDF detector, namely $\sigma = 0.64/\sqrt{E_T}$ in GeV + 0.028. The resolution of the D0 hadron calorimeter is not quite so good: $\sigma \sim 20\%$ for $E_T = 20$ GeV. Moreover the expected resolution of the CMS hadron calorimeter is about twice worse, while the anticipated resolution of the ATLAS detector may be even a bit better: $\sigma \sim 0.5/\sqrt{E_T}$ in GeV + 0.015.

⁹ In the appendix we give the formulae needed to compute exclusive three-jet production in the whole kinematical interval, and not just in the domain of the leading collinear log approximation.

on the helicities, h_i , and the 4-momenta, p_i , of gluons, is given by the so-called dual expansion (see [41] and references therein)

$$M^{h_a, h_b, h_c, h_d, h_e}(p_a, p_b, p_c, p_d, p_e) = \sum \text{Tr}(\lambda_a \lambda_b \lambda_c \lambda_d \lambda_e) \times m(a, b, c, d, e), \quad (\text{A.1})$$

where the sum is over the non-cyclic permutations of a, b, c, d, e . The first factor looks as if all the gluons were emitted from the quark loop, where λ_i are the standard matrices of the fundamental representation of $\text{SU}(3)$, which are normalised as follows:

$$\text{Tr}(\lambda^a \lambda^b) = \frac{1}{2} \delta^{ab}, \quad (\text{A.2})$$

$$[\lambda^a, \lambda^b] = i f_{abc} \lambda^c. \quad (\text{A.3})$$

The colour-ordered subamplitudes, $m(a, b, c, d, e)$, are only functions of the kinematical variables of the process, i.e. the momenta and the helicities of the gluons. They may be written in terms of the products of the Dirac bispinors, that is, in terms of the angular (and square) brackets:

$$\langle ab \rangle = \langle p_a^- | p_b^+ \rangle = \sqrt{|2p_a p_b|} e^{i\phi_{ab}}, \quad (\text{A.4})$$

$$[ab] = \langle p_a^+ | p_b^- \rangle = \sqrt{|2p_a p_b|} e^{i\bar{\phi}_{ab}}, \quad (\text{A.5})$$

where $2(p_a p_b) = s_{ab}$ is the square of the energy of the corresponding pair. If both 4-momenta have positive energy, the phase ϕ_{ab} is given by

$$\cos \phi_{ab} = \frac{p_a^x p_b^+ - p_b^x p_a^+}{\sqrt{p_a^+ p_b^+ s_{ab}}}, \quad \sin \phi_{ab} = \frac{p_a^y p_b^+ - p_b^y p_a^+}{\sqrt{p_a^+ p_b^+ s_{ab}}}, \quad (\text{A.6})$$

with $p_i^+ = p_i^0 + p_i^z$, while the phase $\bar{\phi}_{ab}$ can be calculated using the identity $s_{ab} = \langle ab \rangle [ab]$. Actually the phase ϕ_{ab} is irrelevant in our collinear LO calculations, except for the fact that $\langle ab \rangle = -\langle ba \rangle$ and $[ab] = -[ba]$. However to calculate the $gg \rightarrow ggg$ amplitude beyond LO, and to compute

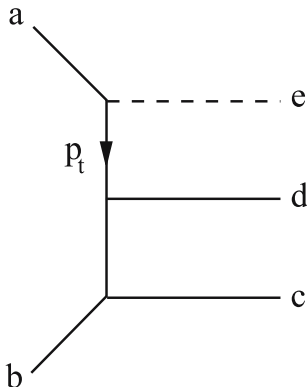


Fig. 6. A schematic diagram of the $gg \rightarrow ggg$ process. The gluon labeled by e is the (soft) third jet

a more precise cross section, based on (A.1) and (A.7), we would have to account for the phases.

Finally, the only non-zero subamplitudes

$$m(a, b, c, d, e) = ig^3 2^{5/2} \frac{\langle IJ \rangle^4}{\langle ab \rangle \langle bc \rangle \langle cd \rangle \langle de \rangle \langle ea \rangle} \quad (\text{A.7})$$

are those which have two helicities of one sign, with the other three of the opposite sign, the so-called maximal helicity violating (MHV) amplitudes. Here g is the QCD coupling ($\alpha_s = g^2/4\pi$). In particular, when $h_a = h_b = -1$ while $h_c = h_d = h_e = +1$, the numerator $\langle IJ \rangle^4 = \langle ab \rangle^4$, i.e. I and J are the only two gluons with the same helicities. If we change the sign of the helicities, then we have simultaneously to replace the $\langle ij \rangle$ brackets by the $[ij]$ brackets. Note that the collinear logarithm in the direction of gluon a comes from the factor $\langle ae \rangle$ (or $\langle ea \rangle$) in the denominator of (A.7). Thus to obtain the LO result it is enough to keep only the permutations where the soft gluon e is close by its nearest neighbour, gluon a .

Note that in the formalism leading to (A.7) all the gluons are considered as incoming particles; that is, the energies of the gluons c, d, e are negative. In the case when one or two momenta in the product $\langle ab \rangle$ have negative energy, the phase ϕ_{ab} is calculated with minus the momenta with negative energy, and then $n\pi/2$ is added to ϕ_{ab} where n is the number of negative momenta in the spinor product.

The three-jet cross section (8) is the square of the matrix element (A.1) calculated using the subamplitudes given by (A.7). In this way, we obtain

$$d\sigma = |M|^2 \frac{\delta^{(4)}(\sum_i p_i)}{64\pi^5 s_{ab}} \prod_j \frac{d^3 p_j}{2E_j}, \quad (\text{A.8})$$

where $i = a, b, c, d, e$ and $j = c, d, e$. To calculate the collinear LO contribution it is enough to keep, in (A.1), only the permutations where the soft gluon e is the nearest neighbour of the incoming gluons a or b . For example, for the case of e collinear to a we need only retain the $m(a, e, b, c, d)$ and $m(a, b, c, d, e)$ subamplitudes, plus the analogous amplitudes with all the permutations of the gluons b, c, d . When we sum over the permutations of gluons b, c, d , and account for the fact that in collinear approximation the 4-vector e_μ is parallel to a_μ , we obtain the exclusive amplitude of high- E_T di-jet production. The factor $\langle ae \rangle$ in the denominator of the subamplitude provides the LO logarithm ds_{ae}/s_{ae} in the cross section.

References

1. M.G. Albrow, A. Rostovtsev, arXiv:hep-ph/0009336
2. V.A. Khoze, A.D. Martin, M.G. Ryskin, Eur. Phys. J. C **23**, 311 (2002)
3. A. De Roeck, V.A. Khoze, A.D. Martin, R. Orava, M.G. Ryskin, Eur. Phys. J. C **25**, 391 (2002)
4. B.E. Cox, AIP Conf. Proc. **753**, 103 (2005) [arXiv:hep-ph/0409144]

5. A.B. Kaidalov, V.A. Khoze, A.D. Martin, M.G. Ryskin, *Eur. Phys. J. C* **33**, 261 (2004)
6. V.A. Khoze, S. Heinemeyer, M.G. Ryskin, W.J. Stirling, M. Tasevsky, G. Weiglein, to be published
7. M.G. Albrow et al., CERN-LHCC-2005-025
8. V.A. Khoze, A.D. Martin, M.G. Ryskin, *Eur. Phys. J. C* **14**, 525 (2000)
9. J.R. Forshaw, arXiv:hep-ph/0508274
10. K. Goulianos, arXiv:hep-ph/0407035
11. K. Goulianos, arXiv:hep-ph/0510035
12. CDF-Run II Collaboration, M. Gallinaro, arXiv:hep-ph/0505159
13. C. Mesropian, arXiv:hep-ph/0510193
14. M. Gallinaro (on behalf of the CDF Collaboration), *Acta Phys. Polon. B* **35**, 465 (2004) [arXiv:hep-ph/0410232], Talk at the XIV International Workshop on Deep Inelastic Scattering, 20–24 April 2006, Tsukuba, Japan
15. K. Terashi, Talk at the XLI Rencontres de Moriond, March 18–25, 2006, Vallee d’Aoste, Italy
16. A.B. Kaidalov, V.A. Khoze, A.D. Martin, M.G. Ryskin, *Eur. Phys. J. C* **21**, 521 (2001)
17. A.B. Kaidalov, V.A. Khoze, A.D. Martin, M.G. Ryskin, *Phys. Lett. B* **559**, 235 (2003)
18. V.A. Khoze, A.D. Martin, M.G. Ryskin, W.J. Stirling, *Eur. Phys. J. C* **35**, 211 (2004)
19. A.D. Martin, A.B. Kaidalov, V.A. Khoze, M.G. Ryskin, W.J. Stirling, *Czech. J. Phys.* **55**, B717 (2005) [arXiv:hep-ph/0409258]
20. V.A. Khoze, A.B. Kaidalov, A.D. Martin, M.G. Ryskin, W.J. Stirling, arXiv:hep-ph/0507040
21. M.G. Albrow, A. Hamilton, presentation at the Workshop on Future of Forward Physics at the LHC, Manchester, December 2005
22. V.A. Khoze, A.D. Martin, M.G. Ryskin, W.J. Stirling, *Eur. Phys. J. C* **38**, 475 (2005)
23. V.A. Khoze, A.D. Martin, M.G. Ryskin, *Eur. Phys. J. C* **19**, 477 (2001)
24. M. Boonekamp, R. Peschanski, C. Royon, *Phys. Rev. Lett.* **87**, 251 806 (2001)
25. C. Royon, arXiv:hep-ph/0601226 and references therein
26. R.B. Appleby, J.R. Forshaw, *Phys. Lett. B* **541**, 108 (2002)
27. V.N. Baier, E.A. Kuraev, V.S. Fadin, V.A. Khoze, *Phys. Rept.* **78**, 293 (1981)
28. F. Bloch, A. Nordsieck, *Phys. Rev.* **52**, 54 (1937)
29. T. Kinoshita, *J. Math. Phys.* **3**, 650 (1962)
30. T.D. Lee, M. Nauenberg, *Phys. Rev. B* **133**, 1549 (1964)
31. J. Monk, A. Pilkington, arXiv:hep-ph/0502077
32. B.E. Cox, J.R. Forshaw, *Comput. Phys. Commun.* **144**, 104 (2002)
33. CDF Collaboration, T. Affolder et al., *Phys. Rev. Lett.* **85**, 4215 (2000)
34. B.E. Cox, A. Pilkington, *Phys. Rev. D* **72**, 094024 (2005)
35. V.A. Khoze, A.D. Martin, M.G. Ryskin, *Eur. Phys. J. C* **18**, 167 (2000)
36. M.A. Kimber, A.D. Martin, M.G. Ryskin, *Phys. Rev. D* **63**, 114027 (2001)
37. G. Watt, A.D. Martin, M.G. Ryskin, *Eur. Phys. J. C* **31**, 73 (2003)
38. V.V. Sudakov, *Sov. Phys. JETP* **3**, 65 (1956) [*Zh. Eksp. Teor. Fiz.* **30**, 87 (1956)]
39. E. Kuraev, V. Fadin, *Sov. J. Nucl. Phys.* **27**, 293 (1987)
40. Y.L. Dokshitzer, V.A. Khoze, A.H. Mueller, S.I. Troyan, in *Basics of perturbative QCD* (Editions Frontières 1991)
41. M.L. Mangano, S.J. Parke, *Phys. Rep.* **200**, 301 (1991)
42. F.E. Low, *Phys. Rev.* **110**, 974 (1958)
43. D.L. Borden, V.A. Khoze, W.J. Stirling, J. Ohnemus, *Phys. Rev. D* **50**, 4499 (1994)
44. Yu.L. Dokshitzer, V.A. Khoze, W.J. Stirling, *Nucl. Phys. B* **428**, 3 (1994)
45. A.D. Martin, R.G. Roberts, W.J. Stirling, R.S. Thorne, *Eur. Phys. J. C* **14**, 133 (2000)
46. V.M. Abazov et al., submitted to *Nucl. Instrum. Methods*, arXiv:physics/0507191, Fermilab-Pub-05/341-E
47. C. Royon, arXiv:hep-ph/0601226 and references therein